As the order number of the element increases the range r_0 and together with it the ratio r_0/r_L decreases (for dia- amd paramagnets r_L is virtually independent of the material) and therefore the effect of the characteristic magnetic field on the character of the energy release becomes weaker. The broken curve 5 was constructed in [6] for the same values of I, E_0, and R as those used for curve 3. The differences in the results are attributable to the fact that for the step size s chosen in [6] ($\Delta E = 0.05E$) the condition that the effective angle of deflection in the magnetic field be limited ($b < 1$ rad) is not satisfied ($b \approx 2.5$ rad).

The calculations showed that as the electron energy E_0 increases the character of the self-consistent distribution of the energy release in range units remains virtually unchanged. This is attributable to the fact that in the energy range studied (1-5 MeV) the ratio r_0/r_L is practically constant.

LITERATURE CITED

1. A. F. Akkerman, Yu. M. Nikitushev, and V. A. Botvin, Monte Carlo Solution of Problems of the Transport of Fast Electrons in Matter [in Russian], Alma-Ata (1972).
2. G. E. Gorelik and S. G. Rozin, Inzh.-Fiz. Zh., 22, No. 6, 1110-1113 (1972).
3. L. I. Rudakov, Fiz. Plazmy, 4, 72-77 (1978).
4. K. Imasaki, S. Miyamoto, and S. Higaki, Phys. Rev. Lett., 43, No. 26, 1937-1940 (1979).
5. R. Miller, Introduction to the Physics of Strong-Current Beams of Charged Particles [in Russian], Moscow (1984).
6. V. I. Boiko, E. A. Gorbachev, and V. V. Evstigneev, Fiz. Plazmy, 9, No. 4, 764-769 (1983).
7. A. S. Roshal', Stimulation of Charged Beams [in Russian], Moscow (1979).

GRADIENT OF THE DISCREPANCY IN THE ITERATIVE SOLUTION OF INVERSE HEAT-CONDUCTION PROBLEMS. III. CALCULATION OF THE GRADIENT USING A CONJUGATE BOUNDARY PROBLEM

O. M. Alifanov and S. V. Rumyantsev

UDC 536.24

The determination of the gradient in the discrepancy functional, which is required for the construction of regularizing gradient algorithms for their solution, is considered for various formulations of nonlinear inverse problems of generalized heat conduction.

In [1], the conditions of the conjugate boundary problems were derived for the formulation of the second and third boundary problems in the case of a quasilinear generalized heat-conduction equation, and formulas were obtained for determining the discrepancy gradient in terms of the conjugate variable. It was assumed that the time dependence of the temperature at one mobile internal point of a one-dimensional spatial region is known as the initial data.

Below, the conjugate problem is brought to a form in which there is no singular term, the conjugate problem is formulated for the case of measurements at the boundary of the region, and expressions are obtained for the discrepancy gradient in measurements at several spatial points and also for other types of boundary conditions of the problem.

As in [1], the gradient of the discrepancy functional $J = \frac{1}{2}\int_0^{\tau_m} [T(d(\tau), \tau) - f(\tau)]^2 d\tau$ with respect to the functions $\xi(x)$, $p_1(\tau)$, $p_2(\tau)$, and the numercial vectors $\bar{\lambda} = \{\lambda_j\}_1^{M_1}$, $\bar{C} = \{C_j\}_1^{M_2}$, $\bar{K} = \{K_j\}_1^{M_3}$, $\bar{g} = \{g_j\}_1^{M_4}$ is considered, for the following conditions

$$CT_\tau = (\lambda T_x)_x + KT_x + g,$$
$$(x, \tau) \in Q_\tau = \{X_1(\tau) < x < X_2(\tau), \quad 0 < \tau < \tau_m\};$$

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 6, pp. 981-986, June, 1987. Original article submitted January 6, 1986.

0022-0841/87/5206-0708 $12.50

$$T(x, 0) = \xi(x); \quad [\alpha_i \lambda T_x + \beta_i T]_{x=X_i(\tau)} = p_i(\tau), \quad i = 1, 2,$$

where

$$\lambda(T) = \sum_{j=1}^{M_1} \lambda_j \varphi_j(T), \quad C(T) = \sum_{j=1}^{M_2} C_j \varphi_j(T),$$

$$K(T) = \sum_{j=1}^{M_3} K_j \varphi_j(T), \quad g(T) = \sum_{j=1}^{M_4} g_j \varphi_j(T);$$

$\varphi_i(T)$ are specified basis functions; $\xi(x) \in L_2[X_1(0), X_2(0)]$; $p_i(\tau)$, $f(\tau) \in L_2[0, \tau_m]$; $X_1(\tau)$, $X_2(\tau)$, $d(\tau)$, $\tau \in [0, \tau_m]$ are piecewise-smooth functions corresponding to the conditions: $X_1(\tau) \leqslant d(\tau)$ when $\alpha_1 \neq 0$, $X_1(\tau) < d(\tau)$ when $\alpha_1 = 0$, $d(\tau) \leqslant X_2(\tau)$ when $\alpha_2 \neq 0$, and $d(\tau) < X_2(\tau)$ when $\alpha_2 = 0$.

Note that, if the desired quantities are the functions $\xi(x)$, $p_i(\tau)$, $i = 1, 2$, the coefficients C, λ, K, and the free term g may be specified to depend not only on T but also on the arguments x, τ; their form may differ from that given above when these quantities are regarded as unknowns.

In [1], for the case where $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, a formulation of the boundary problem conjugate to the problem for an increment in $T(x, \tau)$ is obtained, in the form

$$L^*\psi(x, \tau) = h(\tau)\delta(x - d(\tau)), \quad (x, \tau) \in Q_\tau; \tag{1}$$

$$\psi(x, \tau_m) = 0; \tag{2}$$

$$B_{i\tau}^*\psi|_{x=X_i(\tau)} \equiv \left[(a_1\psi)_x - \psi\left(X_i' + a_2 - \frac{\sigma_i}{\gamma_i} a_1\right)\right]_{x=X_i(\tau)} = 0, \quad i = 1, 2, \tag{3}$$

where

$$L^* = -\frac{\partial}{\partial\tau} - A_{x\tau}^*, \quad A_{x\tau}^*\psi = (a_1\psi)_{xx} - (a_2\psi)_x + a_3\psi,$$

$$a_1 = \frac{\lambda}{C}, \quad a_2 = \frac{1}{C}(2\lambda_x + K), \quad a_3 = \frac{1}{C}(\lambda_{xx} + g_T + K_x - C_\tau),$$

$$\gamma_i = \alpha_i \lambda(X_i(\tau), \tau), \quad \sigma_i = \alpha_i \lambda_x(X_i(\tau), \tau) + \beta_i,$$

$$h(\tau) = T(d(\tau), \tau) - f(\tau).$$

Reducing the Conjugate Problems to a Form Convenient for Numerical Solution

In performing calculations, the problem in Eqs. (1)-(3) must be reduced to a form in which there is no singular term. Assuming that $X_1(\tau) < d(\tau) < X_2(\tau)$, the function

$$\Psi(x, \tau) = \begin{cases} \psi_1(x, \tau), & x \in (X_1(\tau), \ d(\tau)), \\ \psi_2(x, \tau), & x \in (d(\tau), \ X_2(\tau)) \end{cases}$$

is introduced, and it may be shown that the system in Eqs. (1)-(3) is equivalent to the following problem

$$-\psi_{1\tau} - A_{x\tau}^*\psi_1 = 0, \ (x, \tau) \in Q_{1\tau} = \{X_1(\tau) < x < d(\tau), \ 0 < \tau < \tau_m\}; \tag{4}$$

$$-\psi_{2\tau} - A_{x\tau}^*\psi_2 = 0, \ (x, \tau) \in Q_{2\tau} = \{d(\tau) < x < X_2(\tau), \ 0 < \tau < \tau_m\}; \tag{5}$$

$$\begin{gathered} B_{1\tau}^*\Psi|_{x=X_1(\tau)} = B_{2\tau}^*\Psi|_{x=X_2(\tau)} = 0; \\ \psi_1(d(\tau), \tau) - \psi_2(d(\tau), \tau) = 0; \end{gathered} \tag{6}$$

$$\begin{gathered} [(a_1\psi_1)_x - (a_1\psi_2)_x]_{x=d(\tau)} = h(\tau); \\ \Psi(x, \tau_m) = 0. \end{gathered} \tag{7}$$

The solution of Eqs. (1)-(3) is the generalized function $\psi(x, \tau)$, $(x,\tau) \in Q_\tau$, satisfying Eqs. (1)-(3) in the generalized sense, that is

$$\psi \in \Gamma: (\varphi, L^*\psi) = (\varphi, h(\tau)\delta(x - d(\tau))),$$

where Γ is the set of generalized functions satisfying the initial and boundary conditions in Eqs. (2) and (3); $\varphi = \varphi(x, \tau)$ is the basic function, which is understood below to be a real function, continuous together with its derivatives φ_τ, φ_x, and φ_{xx}, and finite in the region Q_τ; (φ, p) is the value of the generalized function p multiplied by the basic function φ.

The condition ensuring differentiability of the operator of the direct problem [1] guarantees continuity of the coefficients of the operator L* and their corresponding derivatives in the region Q_τ. Therefore, differentiating $(a_1\Psi)_x$ as a generalized function [2], the result obtained, in view of Eq. (7), is

$$(a_1\Psi)_{xx} = \{(a_1\Psi)_{xx}\} - h(\tau)\delta(x - d(\tau)),$$

where

$$\{a_1\Psi_{xx}\} = \begin{cases}(a_1\psi_1)_{xx}, & x \in (X_1(\tau), \quad d(\tau)),\\ (a_1\psi_2)_{xx}, & x \in (d(\tau), \quad X_2(\tau))\end{cases}$$

is the regular component of the generalized derivative $(a_1\Psi)_{xx}$. The other derivatives Ψ_τ and $(a_2\Psi)_x$ do not have singular components in view of the condition in Eq. (6) and the continuity of a_2. Therefore, taking account of Eqs. (4) and (5), it is found that $\Psi(x, \tau)$ satisfies Eq. (1) in the generalized sense, i.e., the problem in Eqs. (1)-(3) is equivalent to Eqs. (4)-(7), which is what was to be proven.

Finally, considering the formulation of the inverse problem when the point $x = d(\tau)$ at which $f(\tau)$ is specified coincides with one of the boundaries of the region, it is assumed, for example that $d(\tau) = X_2(\tau)$. Proceeding analogously, it may be shown that in this case the conjugate problem takes the form

$$\psi_\tau + A^*_{x\tau}\psi = 0; \quad (x, \tau) \in Q_\tau; \quad \psi(x, \tau_m) = 0;$$
$$B^*_{1\tau}\psi|_{x=X_1(\tau)} = 0; \quad B^*_{2\tau}\psi|_{x=X_2(\tau)} = h(\tau).$$

Case of Several Measurements

Consider the inverse problem for the equations

$$CT_\tau = (\lambda T_x)_x + KT_x + g,$$
$$(x, \tau) \in Q_\tau = \{d_0(\tau) < x < d_{N+1}(\tau), \ 0 < \tau < \tau_m\} \tag{8}$$

with the boundary conditions

$$T(x, 0) = \xi(x); \tag{9}$$

$$[\alpha_1\lambda T_x + \beta_1 T]_{x=d_0(\tau)} = p_1(\tau); \tag{10}$$

$$[\alpha_2\lambda T_x + \beta_2 T]_{x=d_{N+1}(\tau)} = p_2(\tau), \tag{11}$$

assuming that

$$T(d_n(\tau), \tau) = f_n(\tau), \ n = \overline{1, N}, \ N \geqslant 1, \tag{12}$$

are known dependences, where $d_1 < d_2 < \ldots < d_N$.

The desired quantity may be one or more functions from the set $\bar{u} = \{\xi(x), p_1(\tau), p_2(\tau), \lambda(T), C(T), K(T), g(T)\}$. In determining $\xi(x)$ and (or) $p_i(\tau)$, i = 1, 2, it is assumed that $\lambda = \lambda(T, x, \tau)$, $C = C(T, x, \tau)$, $K = K(T, x, \tau)$, $g = g(T, x, \tau)$.

The conditions of the problem for an increment in the field of $T(x, \tau)$ are identical to the conditions in Eqs. (9)-(11) of [1], with the replacement of $X_1(\tau)$ and $X_2(\tau)$ by $d_0(\tau)$ and $d_{N+1}(\tau)$, respectively.

Writing the functional

$$J(\bar{u}) = \frac{1}{2}\sum_{n=1}^{N}\int_0^{\tau_m}[T(\bar{u}, d_n(\tau), \tau) - f_n(\tau)]^2 d\tau, \tag{13}$$

it is assumed that $d_n(\tau)$, $n = \overline{0, N+1}$ are piecewise-smooth functions. First consider the case when $\alpha_1 \neq 0$, $\alpha_2 \neq 0$ and $d_0(\tau) < d_1(\tau)$, $d_N(\tau) < d_{N+1}(\tau)$.

Following the method of obtaining a conjugate boundary problem outlined in [1], and performing the above reduction to a form without a singular term, the following formulation of this problem is obtained

$$\psi_{n\tau} + A^*_{x\tau}\psi_n = 0, \quad (x, \tau) \in Q_\tau, \quad n = \overline{1, N+1};$$

$$\psi_n(x, \tau_m) = 0, \quad n = \overline{1, N+1};$$

$$\psi_n(d_n(\tau), \tau) - \psi_{n+1}(d_n(\tau), \tau) = 0, \quad n = \overline{1, N};$$

$$[(a_1\psi_n)_x - (a_1\psi_{n+1})_x]_{x=d_n(\tau)} = T(\bar{u}, d_n(\tau), \tau) - f_n(\tau), \quad n = \overline{1, N};$$

$$\left[(a_1\psi_1)_x - \psi_1\left(d_0' + a_2 - \frac{\sigma_1}{\gamma_1}a_1\right)\right]_{x=d_0(\tau)} = 0; \tag{14}$$

$$\left[(a_1\psi_{N+1})_x - \psi_{N+1}\left(d_{N+1}' + a_2 - \frac{\sigma_2}{\gamma_2}a_1\right)\right]_{x=d_{N+1}(\tau)} = 0, \tag{15}$$

where $\gamma_1 = \alpha_1\lambda(d_0(\tau), \tau)$; $\gamma_2 = \alpha_2\lambda(d_{N+1}(\tau), \tau)$; $\sigma_1 = \alpha_1\lambda_x(d_0(\tau), \tau) + \beta_1$; $\sigma_2 = \alpha_2\lambda_x(d_{N+1}(\tau), \tau) + \beta_2$.

The formulas for the components of the gradient of the functional in Eq. (13) in this case take the form

$$J'_\xi(x) = \psi_n(x, 0), \quad d_{N-1}(0) \leqslant x \leqslant d_n(0), \quad n = \overline{1, N+1}; \tag{16}$$

$$J'_{p_1}(\tau) = -\psi_1(d_0(\tau), \tau)\frac{a_1(d_0(\tau), \tau)}{\gamma_1(\tau)}; \tag{17}$$

$$J'_{p_2}(\tau) = \psi_{N+1}(d_{N+1}(\tau), \tau)\frac{a_1(d_{N+1}(\tau), \tau)}{\gamma_2(\tau)}; \tag{18}$$

$$J'_{\lambda_l} = \Phi_0 + \Phi_1 + \Phi_{N+1}, \quad l = \overline{1, M_1}; \tag{19}$$

$$J'_{C_l} = -\sum_{n=1}^{N+1}\int_0^{\tau_m} d\tau \int_{d_{n-1}(\tau)}^{d_n(\tau)} \frac{\psi_n(x, \tau)}{C(x, \tau)} T_\tau(x, \tau)\varphi_l(T(x, \tau))\,dx, \quad l = \overline{1, M_2}; \tag{20}$$

$$J'_{K_l} = \sum_{n=1}^{N+1}\int_0^{\tau_m} d\tau \int_{d_{n-1}(\tau)}^{d_n(\tau)} \frac{\psi_n(x, \tau)}{C(x, \tau)} T_x(x, \tau)\varphi_l(T(x, \tau))\,dx, \quad l = \overline{1, M_3}; \tag{21}$$

$$J'_{g_l} = \sum_{n=1}^{N+1}\int_0^{\tau_m} d\tau \int_{d_{n-1}(\tau)}^{d_n(\tau)} \frac{\psi_n(x, \tau)}{C(x, \tau)} \varphi_l(T(x, \tau))\,dx, \quad l = \overline{1, M_4}, \tag{22}$$

where

$$\Phi_0 = \sum_{n=1}^{N+1}\int_0^{\tau_m} d\tau \int_{d_{n-1}(\tau)}^{d_n(\tau)} \frac{\psi_n(x, \tau)}{C(x, \tau)}\left[T_{xx}(x, \tau)\varphi_l(T(x, \tau)) + T_x^2(x, \tau)\frac{d\varphi_l(T(x, \tau))}{dT}\right]dx,$$

$$\Phi_i = \int_0^{\tau_m} \frac{\psi_i(x, \tau)}{C(x, \tau)} T_x(x, \tau)\varphi_l(T(x, \tau))|_{x=d_i(\tau)}\,d\tau, \quad i = 1, N+1.$$

If the functions at the boundaries of the region $f_0(\tau)=T(d_0(\tau),\tau)$, $f_{N+1}(\tau)=T(d_{N+1}(\tau),\tau)$, are added to the given N dependence $f_n(\tau)$, Eqs. (14) and (15) may be specified with right-hand sides equal to $T(\bar{u}, d_0(\tau),\tau)-f_0(\tau)$ and $T(\bar{u}, d_{N+1}(\tau),\tau)-f_{N+1}(\tau)$, respectively.

Inverse Problem with Boundary Conditions of the First Kind and in a Mixed Formulation

Repeating the entire scheme for obtaining the gradient of the functional in Eq. (13), it may be shown that the following results apply to the formulation of the inverse problem corresponding to the conditions in Eqs. (8)-(12) with $\alpha_1=\alpha_2=0$, $\beta_1=\beta_2=1$; $d_0(\tau)<d_1(\tau)$, $d_N(\tau)<d_{N+1}(\tau)$. The problem for the field increment is

$$v_\tau = a_1 v_{xx} + a_2 v_x + a_3 v + q + \omega, \quad (x,\ \tau)\in Q_\tau; \quad v(x,\ 0) = \Delta\xi;$$
$$v(d_0(\tau),\ \tau) = \Delta p_1(\tau); \quad v(d_{N+1}(\tau),\ \tau) = \Delta p_2(\tau),$$

where $q=\frac{1}{C}[\Delta\lambda T_{xx}+(\Delta\lambda)_x T_x+\Delta K T_x+\Delta g-\Delta C T_\tau]$ is the remainder term.

The conjugate problem is

$$\psi_{n\tau} + A^*_{x\tau}\psi_n = 0, \quad (x,\ \tau)\in Q_\tau, \quad n=\overline{1,\ N+1};$$
$$\psi_n(x,\ \tau_m) = 0, \quad n=\overline{1,\ N+1};$$
$$\psi_n(d_n(\tau),\ \tau) - \psi_{n+1}(d_n(\tau),\ \tau) = 0,\ n=\overline{1,\ N};$$
$$[(a_1\psi_n)_x - (a_1\psi_{n+1})_x]_{x=d_n(\tau)} = T(\bar{u},\ d_n(\tau),\ \tau) - f_n(\tau),\ n=\overline{1,\ N};$$
$$\psi_1(d_0\ (\tau),\ \tau) = \psi_{N+1}(d_{N+1}(\tau),\ \tau) = 0.$$

The formula for the components of the gradient $J'_\xi(x)$, J'_{C_l}, J'_{K_l}, J'_{g_l} are identical to the corresponding Eqs. (16), (20), (21), and (22). The components $J'_{p_1}(\tau)$, $J'_{p_2}(\tau)$, J'_{λ_l} take the form

$$J'_{p_1}(\tau) = [a_1(d_0(\tau),\ \tau)\,\psi_1(d_0(\tau),\ \tau]_x, \tag{23}$$

$$J'_{p_2}(\tau) = -[a_1(d_{N+1}(\tau),\ \tau)\,\psi_{N+1}(d_{N+1}(\tau),\ \tau)]_x, J'_{\lambda_l} = \Phi_0, \quad l=\overline{1,\ M_1}. \tag{24}$$

Finally, consider the mixed boundary formulation of the inverse problem when $\alpha_1 = 0$, $\beta_1 = 1$; $\alpha_2 \neq 0$ and the measurements are specified at the points $d_n(\tau)$, $n = \overline{1,\ N+1}$, while $d_0(\tau) < d_1(\tau)$. In this case, the problem for the field increment is determined by the conditions

$$v_\tau = a_1 v_{xx} + a_2 v_x + a_3 v + q + \omega, \quad (x,\ \tau)\in Q_\tau; \quad v(x,\ 0) = \Delta\xi;$$
$$v(d_0(\tau),\ \tau) = \Delta p_1(\tau); \ [\gamma_2 v_x + \sigma_2 v]_{x=d_{N+1}(\tau)} = \Delta p_2(\tau) + \omega_2.$$

The conjugate problem takes the form

$$\psi_{n\tau} + A^*_{x\tau}\psi_n = 0, \quad (x,\ \tau)\in Q_\tau,\ n=\overline{1,\ N+1};$$
$$\psi_n(x,\ \tau_m) = 0, \quad n=\overline{1,\ N+1};$$
$$\psi_n(d_n(\tau),\ \tau) - \psi_{n+1}(d_n(\tau),\ \tau) = 0, \quad n=\overline{1,\ N};$$
$$[(a_1\psi_n)_x - (a_1\psi_{n+1})_x]_{x=d_n(\tau)} = T(\bar{u},\ d_n(\tau),\ \tau) - f_n(\tau),\ n=\overline{1,\ N};$$
$$\psi_1(d_0(\tau),\ \tau) = 0;$$
$$\left[(a_1\psi_{N+1})_x - \psi_{N+1}\left(d'_{N+1} + a_2 - \frac{\sigma_2}{\gamma_2}a_1\right)\right]_{x=d_{N+1}(\tau)} = T(\bar{u},\ d_{N+1}(\tau),\ \tau) - f_{N+1}(\tau).$$

The components of the gradient are determined here by Eqs. (16), (23), (18), and (20)-(22) and the formula

$$J'_{\lambda_l} = \Phi_0 + \Phi_{N+1}, \quad l=\overline{1,\ M_1}.$$

Note, in conclusion, that the effective computational algorithms for solving conjugate boundary problems may be obtained on the basis of transforming the initial region with mobile boundaries to a rectangular region with simultaneous straightening of the lines at which the input data $f_n(\tau)$ are specified. This transformation is made by means of the variable substitution

$$y_n = \frac{x - d_{n-1}(\tau)}{d_n(\tau) - d_{n-1}(\tau)}, \quad n = \overline{1, N}; \quad t = \tau.$$

LITERATURE CITED

1. O. M. Alifanov and S. V. Rymyantsev, Inzh.-Fiz. Zh., 52, No. 4, 668-675 (1987).
2. V. S. Vladimirov, Equations of Mathematical Physics, Marcel Dekker (1971).

RESOLVING POWER OF THE ITERATION METHOD OF SOLVING INVERSE HEAT-CONDUCTION BOUNDARY-VALUE PROBLEMS

S. L. Balakovskii

UDC 536.24

A quantitative estimate is obtained for the range of frequencies entering in the boundary condition that is restorable by using a solution of the inverse problem by an iteration method.

Methods of solving inverse heat-conduction problems (IHCP) should possess smoothing properties that would not cause fluctuations of the solution. Such smoothing is assured, for instance, because of the natural step regularization [1], the introduction of extremal formulations of the IHCP and so-called stabilizing functionals [1, 2]. Regularized algorithms are used to seek the solution in a set of functions possessing a definite degree of smoothness, and suppress high frequencies in the parameters being recovered. However, if the fluctions in the desired characteristics are physical in nature, then the viscosity properties of the regularized algorithms do not permit detection of the fine structural features of the solution.

Therefore, when solving the IHCP a situation must be met when the "noninertial" unregularized algorithms pass high frequencies, but because of incorrectness there is no possibility of clarifying the physical component among them and regularization does not afford such a possibility because it filters high harmonics independently of their origin. There, there-, fore, arises the problem of determining the range of frequencies in the restorable parameters as a preliminary step in the selection of methods of raising the accuracy of solving the IHCP, especially in the case of complex behavior of the desired functions. Increasing the measurement accuracy, taking account of a priori information [1], rational placement of the temperature sensors in the object under investigation with application of the Fisher information matrix [3] can be such methods.

Two reasons for suppression of the high frequencies are represented essential for the solution of inverse problems: the smoothing action of the heat-conduction operator and the discretization of the continuously formulated problem.

To obtain quantitative estimates of the passband, we consider the model of a semiinfinite body with thermal diffusivity coefficient a. As is noted in [1], the smoothing action of the heat conduction operator can be estimated by giving the change in body surface temperature according to a sinusoidal law $T_W = T_0 \sin(\omega\tau)$. Then after a certain time the temperature at a depth h will also be described by a sinusoid [4]

$$T(h, \tau) = T_h \sin(\omega\tau - \varphi), \tag{1}$$

where φ is a certain phase difference.

The amplitude of the oscillations T_h is defined as follows

$$T_h = T_0 \exp\left(-\sqrt{\frac{\omega}{2a}}\, h\right). \tag{2}$$

V. V. Kuibyshev Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 6, pp. 986-990, June, 1987. Original article submitted March 4, 1986.